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14. ABSTRACT
The research supported by the AFOSR considered large-scale mathematical programming problems that can be split into two stages. These problems are of particular interest in setting involving substantial data uncertainty. In this case, a first-stage set of decisions may determine long-term strategic decisions, while a second-stage set of tactical-level decisions are made in response to both the first-stage decisions and the observation of random data. This may involve, for instance, the deployment of supply bases in a field (in the first-stage) along with contingency routing plans in the face of operational needs (where the operational needs are the random data, and the routing decisions are the logistical decisions). The work done under this proposal expands the scope and size of problems that can be solved under this paradigm, with particular attention paid to those problems of interest to the AFOSR.

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Final Report: Variable Expansion Techniques for Decomposable Optimization Problems

AFOSR Grant FA9550-08-1-0189

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Abstract

The research supported by the AFOSR considered large-scale mathematical programming problems that can be split into two stages. These problems are of particular interest in setting involving substantial data uncertainty. In this case, a first-stage set of decisions may determine long-term strategic decisions, while a second-stage set of tactical-level decisions are made in response to both the first-stage decisions and the observation of random data. This may involve, for instance, the deployment of supply bases in a field (in the first-stage) along with contingency routing plans in the face of operational needs (where the operational needs are the random data, and the routing decisions are the logistical decisions). The work done under this proposal expands the scope and size of problems that can be solved under this paradigm, with particular attention paid to those problems of interest to the AFOSR.

1   Introduction

This section covers a summary of the nature of research proposed in the original document. Bi-level optimization problems have received much attention in the last decade due to the emerging importance of research in fields like stochastic programming and network interdiction. Also, because many of these algorithms are amenable to parallel implementations, the advent of efficient parallel computing grids has allowed modern bi-level techniques to solve problems heretofore regarded as intractable. In other scenarios, solving problems in two stages allows effort to be conserved by avoiding the explicit solution of problems by mathematical programming. The presence of integer-valued variables induces computational challenges that tend to confound integer programming solvers.

As a result, many problems arising in stochastic asset deployment, network interdiction,

survivable network design, facility location, logistics, and so on, cannot yet be optimized in real-world settings. The research program funded addressed this shortcoming by developing techniques to attack one of the primary culprits behind these computational difficulties: the weakness of a class of cutting planes employed in bi-level algorithms.

The theme of the research program, in general, is the solution of multi-level optimization problems in which integer variables appear in both stages of the problem. In particular, we have explored a surprising property of decomposition algorithms observed by the investigator. We noted that several alternative optimal solutions exist to the dual of the second-stage problems, each resulting in a different Benders inequality. However, it turns out that it is possible to reformulate the master problem to avoid this "exponential cut" difficulty. We have shown that this master problem reformulation technique can dramatically reduce the number of iterations required by Benders decomposition to converge to an optimal solution.

Furthermore, we have investigated a class of two-stage integer programming problems in which integers exist in both stages of the problem. These problems are very difficult to solve, because there does not exist a dual problem that can be readily exploited for the second-stage problems. More to the point, the second-stage problem is not "convex," in the sense that there is no convex function that captures the behavior of the second-stage objective as a function of the first-stage variables. However, by examining a polynomially-expanded first-stage variable space, we were able to show how a class of such problems can indeed be solved in a cutting-plane decomposition fashion, thus enabling solution methods for the first time to very complex problems.

Finally, we showed how the expansion of first-stage variables allows certain problems to be solved by decomposition, which are normally solved as single-stage (monolithic) problems. This is of particular interest, because there exist many problems for which integer programming techniques (or more generally, nonconvex mathematical programming methods) are well suited only for a portion of the problem's constraints. Thus, splitting a problem into two stages such that mathematical programming is applied to one part, while an implicit enumeration algorithm (e.g., dynamic programming) is applied to another, can yield attractive algorithms.

The remainder of this report classifies the problems studied here into one of three groups, as discussed in the following section. The data from the proposed research is given in the final section.

2 Discussion of Research Accomplishments

The introduction and summary above detail the different viewpoints that comprise the funded research program. In this section, we give more details on how each individual paper described in Section 3 is tied to the research project.

2.1 Exponential Cut Capture

This section regards papers that have have been published in which a polynomial expansion in the first-stage variables allows the generation of cutting planes that are far stronger than cuts generated in the original variable space. In particular, these papers show that an exponential number of cuts in the expanded variable space are implied by cuts in the expanded variable space.

Taskin et al. (see reference 7) consider the edge-partition problem, which is a graph theoretic problem arising in the design of Synchronous Optical Networks. The deterministic edge-partition problem considers an undirected graph with weighted edges, and simultaneously assigns nodes and edges to subgraphs such that each edge appears in exactly one subgraph, and such that no edge is assigned to a subgraph unless both of its incident nodes are also assigned to that subgraph. Additionally, there are limitations on the number of nodes and on the sum of edge weights that can be assigned to each subgraph. Our research group examined a stochastic version of the edge-partition problem in which we assign nodes to subgraphs in a first stage, realize a set of edge weights from a finite set of alternatives, and then assign edges to subgraphs. Our two-stage cutting-plane approach is complicated by the fact that integer variables appear in both stages of the problem. We are able, though, to give a valid inequality generation scheme in which variable expansion provides a substantial decrease in computational time over a more standard decomposition approach. (However, as we will show below, there are still further difficulties with this approach that must be overcome with a method that we investigated in a subsequent phase of our investigation.)

Smith et al. (reference 8) consider a scenario in which two firms are determining which products to develop and introduce to a market. In this problem, there exists a finite set of products and market segments, and each market segment has a priority list of products that they will purchase if available. The firms play a Stackelberg game, in which the leader acts first to introduce a set of products, after which the follower firm responds with its own set of products. In a military setting, one might imagine that the leader and follower are both establishing bases to control as much area as possible. The "products" brought to

market are physical base locations. The "segments" would be the key areas of interest to the competitors.

The goal of the leader is to maximize its profit, assuming that the follower will attempt to minimize the leader's profit. This problem is formulated as a multi-stage integer programming problem amenable to decomposition techniques. Using this formulation, we develop three variations of an exact mathematical programming method for this problem, the most effective of which (by a large margin) employed the variable expansion techniques prescribed by this proposal. In this setting, variable expansion takes a different form by using a variable priority-ordering strategy. (We have since also seen this specific technique gainfully employed by our colleagues in other security settings.)

2.2 Integer-Integer Second-Stage Problems

This section gives research highlights of papers that involve integer variables in the second-stage of bilevel optimization problems. Most of the approaches entailed in these works regard a quadratic expansion of the first-stage variable space, as prescribed by the Reformulation-Linearization Technique of Sherali and Adams. The reformulation of these problems permits valid inequalities that are sufficient to solve a certain class of integer two-stage problems to optimality within the cutting-plane paradigm.

Foundations

Foundations in this area are established by Sherali and Smith (references 6 and 14), by Hartman et al. (reference 11), and by Smith et al. (reference 13). Sherali and Smith (reference 6) considered a class of two-stage stochastic risk management problems. Here, a decision-maker determines a set of binary first-stage decisions, after which a random event from a finite set of possible outcomes is realized. Depending on the realization of this outcome, a set of continuous second-stage decisions must then be made that attempt to minimize some risk function. Our approach is to consider a hierarchy of multiple risk levels along with associated penalties for each possible scenario. The overall objective function thus depends on the cost of the first-stage decisions, plus the expected second-stage risk penalties. This contribution develops a mixed-integer 01 programming model and adopts an automatic convexification procedure using the Reformulation-Linearization Technique referenced earlier in this report, to recast the problem into a form that is amenable to applying Benders partitioning approach. As a principal computational expedient, we show how the reformulated higher-dimensional

Benders subproblems can be efficiently solved via certain reduced-sized linear programs in the original variable space. This particular development is crucial in the development of the Shen and Smith (reference 12) paper discussed below.

Sherali and Smith (reference 14) examine the generation of disjunctive cuts for 0-1 mixed-integer programs. The approach conducts a partial exploration of the branch-and-bound tree for these problems using quick Lagrangian primal and dual updates. In doing so, cutting planes, which are critical for two-stage integer programming problems, can quickly be obtained by approximate solutions of the branch-and-bound subproblems. Alternative cut generation strategies are given that are based on formulating different disjunctions and adopting various choices of normalization techniques. By contrast, Hartman et al. (reference 11) show how the value functions of a class of pseudopolynomially-solvable problems can be projected onto a subspace of variables. (In this sense, this paper prescribes a variable *compression* algorithm rather than expansion.) These functions are not necessarily convex, but facets to the convex hull of these value functions can be employed within cutting-plane approaches for the problems.

Smith et al. (reference 13) consider the problem of computing an optimal *branch decomposition* of a graph. Branch decompositions have proven to be useful in solving many NP-hard problems, such as the traveling salesman, independent set, and ring routing problems, by means of combinatorial algorithms that operate on branch decompositions. We develop an implicit enumeration algorithm for the optimal branch decomposition problem and examine its performance on a set of classical graph instances. The goal of this research, as relevant to the proposal given here, is to represent some graphs via their optimal branch decompositions, which would enable the quick solution of problems that would otherwise be strongly NP-hard. With this branch decomposition in place, we would then possibly be able to solve very difficult problems by decomposition that have thus far eluded practical solution methods.

Problems

Two of the PI's students have been actively engaged in examining problem classes that fall under this subsection's problem category. John Penuel and the PI considered several classes of problems, one of which was honored with a best track paper award at the 2008 Industrial Engineering Research Conference. In this problem (reference 10), Penuel and Smith study a stochastic scenario-based facility location problem arising in situations when facilities must first be located, then activated in a particular scenario before they can be used to satisfy scenario demands. Unlike typical facility location problems, fixed charges arise in

the initial location of the facilities, and then in the activation of located facilities. The first-stage variables in our problem are the traditional binary facility-location variables, while the second-stage variables involve a mix of binary facility-activation variables and continuous flow variables. Traditional Benders decomposition is not applicable for these problems due to the presence of the second-stage integer activation variables. Instead, we derive cutting planes tailored to the problem under investigation from recourse solution data. These cutting planes are derived by solving a series of specialized shortest path problems based on a modified residual graph from the recourse solution, and are tighter than the general cuts established by Laporte and Louveaux for two-stage binary programming problems. This is a unique approach to most of the other approaches given in this proposal.

Another class of problems multi-stage integer programming problems investigated by Penuel and Smith are found in reference 15. This paper regards the problem of using synchronous mobile agents to decontaminate the nodes of a graph given a spreading contamination. After a contamination is detected in the graph, agents move along graph edges to eliminate the contamination. At the same pace, the contamination spreads to adjacent nodes. We consider several variants of the problem based on capabilities of the mobile agents, which give rise to problems of various difficulty in multiple stages. The most complicated of these problems seeks to establish the best positions for some predetermined number of agents to minimize expected cleaning time, given probability estimates for potential contaminations starting locations. The problems are all strongly NP-hard, as we show, and the two-stage integer programming formulations can be addressed by cutting-plane techniques as described above.

Ms. Siqian Shen (graduating Summer 2011 with the support of the AFOSR) has co-authored a series of four papers with the PI. In reference 12, Shen et al. consider a class of two-stage stochastic optimization problems arising in the protection of vital arcs in a critical path network. A project is completed after a series of dependent tasks are all finished. Shen et al. analyze a problem in which task finishing times are uncertain but can be insured a priori to mitigate potential delays. A decision maker must trade off costs incurred in insuring arcs with expected penalties associated with late project completion times, where lateness penalties are assumed to be lower semicontinuous nondecreasing functions of completion time. We provide decomposition strategies to solve this problem with respect to either convex or nonconvex penalty functions. In particular, for the nonconvex penalty case, we employ the reformulation-linearization technique to make the problem amenable to solution via Benders decomposition (see also the Sherali and Smith foundational work referenced above). We also consider a chance-constrained version of this problem, in which the probability of completing a project on time is sufficiently large.

A pair of node deletion problems (references 17 and 18) are also considered by Shen and

Smith, in which integer variables appear in both stages. Unlike in the previously discussed problems in this section, the two-stage problems here are interdiction models involving opposing players in competition with one another. These papers analyze the problem of maximizing the disconnectivity of undirected graphs by deleting a subset of their nodes. We consider three metrics that measure the connectivity of a graph: The number of connected components (which we attempt to maximize), the largest component size (which we attempt to minimize), and the minimum cost required to reconnect the graph after the nodes are deleted (which we attempt to maximize). For the first two problems, we demonstrate that the problem is polynomially solvable on special structures such as trees and series-parallel graphs. Moreover, for so-called k-hole subgraphs, the problems are also polynomial given fixed values of k. Then, for general problems, we formulate all three problems as mixed-integer programs, and then study valid inequalities for the first two connectivity objectives by examining intermediate dynamic programming solutions to k-hole subgraphs.

Shen and Smith (reference 19) consider another two-stage optimization problem that integrates network design and broadcast domination decisions. Given an undirected graph, a feasible broadcast domination is a set of nonnegative integer powers f_i assigned to each node i, such that for any node j in the graph, there exists some node k having a positive f_k-value whose shortest distance to node j is no more than f_k. The cost of a broadcast domination solution is the sum of all node power values. The network design problem constructs edges that decrease the minimum broadcast domination cost on the graph. The overall problem that we consider in reference 19 minimizes the sum of edge construction costs and broadcast domination costs.

This problem is particularly interesting in the current setting, because the broadcast domination problem is polynomially solvable on unweighted graphs (although it is NP-hard on weighted graphs). However, there is as of yet no polyhedral description of the broadcast domination polytope, and no method for parametrically solving the broadcast domination problem as a function of network design. Hence, the problem "acts" as though it were hard for the purposes of our research. The design problem is (as we prove) NP-hard in the strong sense, even on unweighted graphs. We then propose a decomposition strategy, which iteratively adds valid inequalities based on optimal broadcast domination solutions corresponding to the first-stage network design solutions. We demonstrate that our decomposition approach is computationally far superior to the solution of a single large-scale mixed-integer programming formulation.

2.3   Monolithic versus Bilevel Formulations

Here, we describe papers that reformulate first-stage variables to allow the splitting of one-stage (or monolithic) programming problems into two-stage problems. The pattern employed is that first-stage problems can be solved by mixed-integer programming techniques, while further stages can be solved in a manner more amenable to solution, such as implicit enumeration or dynamic programming.

Recall the edge partition problem studied by Taskin et al. above in reference 7. (Z. Caner Taskin was supported by the AFOSR, and graduated with his Ph.D. under the PI's supervision in 2009. Dr. Taskin's dissertation was awarded 1st prize in an international dissertation competition as noted in Section 3.) As a part of this study, we examined computational difficulties associated with the expanded-variable cutting plane algorithm. The problems that arose were due to the number of times that a master integer programming problem needed to be solved, which served as the bottleneck for this algorithm. As an alternative, we prescribed a hybrid integer programming/constraint programming algorithm, after redefining variables in the first phase of the algorithm. This approach was the first one capable of solving realistic-sized test instances within practical computational limits.

Taskin et al. also studied a problem in medical operations, using a similar solution paradigm as studied above. Our research group considered a problem dealing with the efficient delivery of intensity modulated radiation therapy (IMRT) to individual patients. We study the problem in which an intensity profile is represented as a nonnegative integer matrix; an aperture is represented as a binary matrix whose ones appear consecutively in each row. A feasible decomposition is one in which the original desired intensity profile is equal to the sum of a number of feasible binary matrices multiplied by corresponding intensity values. To most efficiently treat a patient, we wish to minimize a measure of total treatment time, which is given as a weighted sum of the number of apertures and the sum of the aperture intensities used in the decomposition. We develop the first exact algorithm capable of solving real-world problem instances to optimality within practicable computational limits, using a combination of integer programming decomposition and combinatorial search techniques. We demonstrate the efficacy of our approach on a set of 25 test instances derived from actual clinical data and on 100 randomly generated instances.

2.4   Miscellaneous

Research conducted by the PI during this period that is not related to the proposed area is omitted from this report. However, there are six papers of interest to the AFOSR that have been conducted during this period, and are worth noting given their relevance to the AFOSR mission. They are briefly mentioned here. Garg and Smith (reference 1) consider the design of "survivable" networks, which serve to transmit multicommodity flows across capacitated networks, even when certain sets of network infrastructure simultaneously fails. This is a robust design in the sense that the network operates feasibly in each of the enumerated possible failure combinations, but is stochastic in the sense that an expectation is taken with respect to cost.

In references 2 and 3, we consider robust routing patterns, usually inspired in data networks where communication occurs across working and protection routes. The problem is to design arc-disjoint routes between an origin and a destination, at minimum cost, subject to the probability of at least one path surviving in whole is sufficiently large. A branch-and-cut-and-price algorithm is employed to solve this problem, where the branching takes on a continuous reformulate-and-convexify strategy.

Reference 4 is also an offshoot of this research in which a network design strategy is sought for evacuation networks. Congestion considerations in this evacuation network, along with the unknown congestion timing and travel times on routes, substantially complicate this problem. Moreover, expected evacuation time is not an appropriate measure, because the correct goal for such problems should be the maximization of individuals evacuated within a certain time limit, or subject to another viable risk measure. In this sense, the research is an important precursor to the Shen, Smith, and Ahmed work in reference 12. The two-stage approach designs an evacuation tree in a first phase, examines routing behavior in the second stage, and passes this information back to the first phase to adjust and further optimize the evacuation tree.

Reference 5 is related to references 17 and 18, in which critical nodes are identified to the communication in a network. In references 17 and 18, the nodes are targeted for deletion, after which the graph is shattered into several pieces. In 5, the idea is to force a malicious entity to limit the range and scope of its attacks in a communication network. Finally, reference 16 examines decomposition techniques in the design of wireless networks. Routing is prescribed to maximize the lifetime of batteries in these networks. The long-term ongoing research in this program will be to expand the scope of wireless routing problems that can be solved within realistic time limits. These problems tend to be bilevel, and have integer

variables in both stages, making them ideal for the application of the research supported by the AFOSR.

3   Data on Accomplishments

The following is a list of papers submitted or accepted to refereed journals with AFOSR support. Note that papers 1–14 have been accepted (or appear), while the other papers are in various stages of review.

1. Garg, M. and Smith, J.C., "Models and Algorithms for the Design of Survivable Networks with General Failure Scenarios," *Omega*, 36(6), 1057-1071, 2008.

2. Andreas, A.K., Smith, J.C., and Küçükyavuz, S., "Branch-and-Price-and-Cut Algorithm for Solving the Reliable h-paths Problem," *Journal of Global Optimization*, 42(4), 443-466, 2008.

3. Andreas, A.K. and Smith, J.C., "Mathematical Programming Algorithms for Two-Path Routing Problems with Reliability Constraints," *INFORMS Journal on Computing*, 20(4), 553-564, 2008.

4. Andreas, A.K. and Smith, J.C., "Decomposition Algorithms for the Design of a Non-simultaneous Capacitated Evacuation Tree Network," *Networks*, 53(2), 91- 103, 2009.

5. Mofya, E.C. and Smith, J.C., "Algorithms for the Generalized Minimum Filter Placement Problem on Tree Structures," *INFORMS Journal on Computing*, 21(2), 322-332, 2009.

6. Sherali, H.D. and Smith, J.C., "Two-Stage Stochastic Risk Threshold and Hierarchical Multiple Risk Problems: Models and Algorithms," *Mathematical Programming*, Series A, 120(2), 403-427, 2009.

7. Taskin, Z.C., Smith, J.C., Ahmed, S., and Schaefer, A.J., "Cutting Plane Algorithms for Solving a Robust Edge-Partition Problem," *Discrete Optimization*, 6, 420-435, 2009.

8. Smith, J.C., Lim, C., and Alptekinoglu, A., "Optimal Mixed-Integer Programming and Heuristic Methods for a Bilevel Stackelberg Product Introduction Game," *Naval Research Logistics*, 56(8), 714-729, 2009.

9. Taskin, Z.C., Smith, J.C., Romeijn, H.E., and Dempsey, J.F., "Optimal Multileaf Collimator Leaf Sequencing in IMRT Treatment Planning," *Operations Research*, 58(3), 674-690, 2010.

10. Penuel, J., Smith, J.C., Yuan, Y. "An Integer Decomposition Algorithm for Solving a Two-Stage Facility Location Problem with Second-Stage Activation Costs," *Naval Research Logistics*, 57(5), 391-402, 2010.

11. Hartman, J.C., Büyüktahtakın, I.E., and Smith, J.C., "Dynamic Programming Based Inequalities for the Capacitated Lot-Sizing Problem," to appear in *IIE Transactions*.

12. Shen, S., Smith, J.C., and Ahmed, S., "Expectation and Chance-constrained Models and Algorithms for Insuring Critical Paths," to appear in *Management Science*.

13. Smith, J.C., Ulusal, E., and Hicks, I.V., "A Combinatorial Optimization Algorithm for Solving the Branchwidth Problem," to appear in *Computational Optimization and Algorithms*.

14. Sherali, H.D. and Smith, J.C., "Higher-Level RLT or Disjunctive Cuts Based on a Partial Enumeration Strategy for 0-1 Mixed-Integer Programs," to appear in *Optimization Letters*.

15. Penuel, J. and Smith, J.C., "Models and Complexity Analysis for the Graph Decontamination Problem with Mobile Agents," in revision for *Networks*.

16. Behdani, B., Yun, Y., Smith, J.C., and Xia, Y., "Decomposition Algorithms for Maximizing the Lifetime of Wireless Sensor Networks with Mobile Sinks," revision submitted to *Computers and Operations Research*.

17. Shen, S. and Smith, J.C., "Polynomial-Time Algorithms for Disconnecting Trees and Series-Parallel Graphs under Component Connectivity Metrics," in revision for *Networks*.

18. Shen, S., Smith, J.C., and Goli, R. "Exact Interdiction Models and Algorithms for Disconnecting Networks via Node Deletions," submitted to *Discrete Optimization*.

19. Shen, S. and Smith, J.C., "A Decomposition Approach for Solving a Broadcast Domination Network Design Problem," submitted to *Annals of Operations Research*.

These publications are also peer-reviewed, and appear in conference proceedings and edited books or encyclopedias. The final item listed has been submitted for publication but has not yet finished its review process.

20. Smith, J.C. and Penuel, J., "Solving a Two-Stage Facility Location Problem with Second-Stage Activation Costs," 2008 IERC Conference Proceedings, Vancouver, CA, 2008.

21. Yun, Y., Xia, Y., Behdani, B., and Smith, J.C., "Distributed Algorithm for Lifetime Maximization in Delay-Tolerant Wireless Sensor Network with Mobile Sink," 49th IEEE Conference on Decision and Control, Atlanta, GA, 2010.

22. Smith, J.C., "Basic Interdiction Models," In: Encyclopedia of Operations Research and Management Science (edited by J. Cochran), Wiley, Hoboken, NJ, 2011.

23. Smith, J.C. and Ahmed, S., "Introduction to Robust Optimization," In: Encyclopedia of Operations Research and Management Science (edited by J. Cochran), Wiley, Hoboken, NJ, 2011.

24. Smith, J.C. and Sonuc, S., "An Introduction to Integer and Large-Scale Linear Optimization," In: Wireless Network Design: Optimization Models and Solution Procedures (edited by J.L. Kennington, E.V. Olinick, and D. Rajan), 2011.

25. Prince, M., Geunes, J., and Smith, J.C., "Optimizing Exclusivity Agreements in a Three-Stage Procurement Game," 2011 IERC Conference Proceedings, Reno, NV, 2011.

26. Smith, J.C., Prince, M., and Geunes, J., "Modern Network Interdiction Problems and Algorithms," Submitted for possible publication in: Handbook of Combinatorial Optimization, 2nd Edition, (edited by P.M. Pardalos, D.-Z. Du, and R. Graham), 2011.

The PI has received the following academic awards during the period of performance.

1. 2010 Hamed K. Eldin Outstanding Young Industrial Engineer in Education Award
2. 2009 IIE Operations Research Division Teaching Award
3. 2008 IERC Best Logistics and Inventory Track Paper Award

The following Ph.D. students have received their degrees during this period, with the support of the AFOSR.

1. John Penuel, Decomposition Algorithms for Two-Stage Stochastic Integer Programming, August 2009 August 2009.

2. Z. Caner Taskin, Algorithms for Solving Multi-Level Optimization Problems with Discrete Variables at Multiple Levels, August 2009.

3. I. Esra Buyuktahtakin (co-chair), Mixed Integer Programming Approaches to Lot-Sizing and Asset Replacement Problems, August 2009.

Note that Dr. Taskin's dissertation, supported in part by this funding, won the **Pritsker Doctoral Dissertation Award** sponsored by the Institute of Industrial Engineers. This award annually honors the top dissertation in the field of Industrial Engineering and is a substantial honor. Moreover, current Ph.D. students Behnam Behdani, Sibel Sonuc, Siqian Shen, Kelly Sullivan, and Mike Prince (co-chair) have worked on portions of this research. Their graduation dates range from 2011 from 2013.

The PI has given the following presentations on AFOSR-related research during the reporting period. These are in addition to those seminars given by the Ph.D. students supported by this grant.

1. "An Integer Decomposition Algorithm for Solving a Two-Stage Integer Facility Location Problem with Second-Stage Activation Costs," Invited Lecture, INFORMS Optimization Society, March 2008, Atlanta, GA.

2. "Optimal Insurance of PERT/CPM Arcs with Uncertain Task Durations", Invited Lecture, Industrial Engineering Research Conference, May 2008, Vancouver, CA.

3. "Solving a Two-Stage Facility Location Problem with Second-Stage Activation Costs", Invited Lecture, Industrial Engineering Research Conference, May 2008, Vancouver, CA.

4. "Defense Against K-best Interdiction Attacks," Invited Lecture, INFORMS 2008 Conference, October 2008, Washington, DC.

5. "Leaf Sequencing Algorithms in Radiation Treatment Planning," Invited Lecture, Rice University, December 2008, Houston, TX.

6. "New Approaches to Solving Bi-Level Problems," Invited Lecture, University of South Florida, March 2009, Tampa, FL.

7. "Variable Expansion and Hybrid Optimization Techniques for Solving Bi-Level Problems," Invited Lecture, University of Michigan, March 2009, Ann Arbor, MI.

8. "Cutting Plane Algorithms for Solving a Robust Edge-partition Problem," Invited Lecture, International Symposium on Mathematical Programming, August 2009, Chicago, IL.

9. "New Product Introduction against a Predator: A Bilevel Mixed-Integer Programming Approach," Invited Lecture, University of Maryland, October 2009, College Park, MD.

10. "Two-Stage Facility Location Problem with Second-Stage Activation Costs," Invited Lecture, INFORMS 2009 Conference, October 2009, San Diego, CA.

11. "Variable Expansion and Hybrid Optimization Techniques for Solving Bi-Level Problems," Invited Lecture, University of Pittsburgh, November 2009, Pittsburgh, PA.

12. "Introduction to Robust Optimization," Invited Lecture, Workshop on Combinatorial Optimization under Uncertainty, November 2009, Banff, Canada.

13. "A Mixed-Integer Nonlinear Programming Algorithm for Insuring Critical Paths," Invited Lecture, 2nd LANCS Workshop on Discrete and Non-linear Optimisation, March 2010, Southampton, UK.

14. "Variable Expansion and Hybrid Optimization Techniques for Solving Bi-Level Problems," Invited Lecture, University of Edinburgh, School of Mathematics, April 2010, Edinburgh, UK.

15. "A Decomposition Approach for Insuring Critical Paths," Invited Lecture, University of Edinburgh Business School, June 2010, Edinburgh, UK.

16. "A Decomposition Approach for Insuring Critical Paths," Invited Lecture, University of Warwick, Centre for Discrete Mathematics and its Applications, June 2010, Coventry, UK.

17. "Expectation and Chance-constrained Models and Algorithms for Insuring Critical Paths," Invited Lecture, EURO 2010 Conference, July 2010, Lisbon, Portugal.

18. "A Combinatorial Optimization Algorithm for Solving the Branchwidth Problem," Invited Lecture, INFORMS 2010 Conference, October 2010, Austin, TX.

19. "Searching and Hiding on Networks," Tutorial Lecture, INFORMS 2010 Conference, October 2010, Austin, TX.